Expected Path Length for Angle and Distance-based Localized Routing

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***Abstract*—We give analytical solutions for the expected path length in ad-hoc wireless networks employing distance-based or angle-based based greedy routing schemes. Such schemes underlay many practical routing protocols for ad-hoc wireless networks in which nodes are aware of their geographical locations and can use that information to simplify (localize) the routing rules. As the two paradigms, i.e., minimizing the distance towards the destination, and minimizing the angular deviation from the straight path towards the destination, lie at the heart of practically all location-based routing protocols, our solutions can be viewed as generic and applicable to a variety of realistic schemes.**

I. Introduction

One of the fundamental problems of wireless ad hoc networking is the scalability of the routing information, including its acquisition and storage, to the network size. In wireless networks, the acquisition component scales poorly due to the inherent bandwith limitations of the wireless environment. Moreover, many applications of wireless networks (notably, wireless sensor networks—WSN) are based on nodes with limited resources, which issue affects both acquisition and storage. To eliminate the painful cost of discovering and maintaining detailed routing information at the nodes, a family of memoryless/stateless location-based routing schemes has been proposed. In these protocols, nodes rely solely on the knowledge of their location (which can be established, e.g., via GPS), the location of their neighbors, and the location of the destination. Among the popular and generic solution in this category are the *Greedy* [1], [2] and *Compass* [3] routing schemes representing two natural *localizations* of the global objective which is bringing the packet to its final destination. With the greedy approach, the forwarding node selects the next-hop node as that neighbor that will minimize the packet's residual Euclidean distance to the destination, whereas with the compass scheme, the next-hop neighbor is chosen as to minimize its angular deviation from the straight line connecting the forwarding node to the destination.

Formally, both those paradigms (in their simple generic incarnations) suffer from problems resulting in their inability to deliver all packets even if the network is connected. With Greedy, the packet may hit a *local maximum* (a cul-de-sac) from which no neighbor can offer a shorter distance to the destination than the current node. One can easily see that it makes no sense to forward packets "backwards" without adding some extra rules to the protocol, as that will immediately result in loops. With Compass, such loops are explicit: one can show configurations of nodes whereby the objective of minimizing the angle towards the destination will result in vicious circles, i.e., local and global progress need not be compatible.

However, despite these shortcomings, the simple schemes are difficult to outperform on reasonably dense random networks [4]. This means that they provide a solid base for practical solutions, which may only occasionally need a (statistically minor) *fix* to circumvent a "degenerate" neighborhood. Consequently, general results obtained for pure Greedy and Compass are likely to apply to many statistically relevant networks, for as long as their actual schemes do not attempt to improve too much upon those generic paradigms. This assumption has been the motivation behind our work.

II. Previous work

The problem is defined as follows. Suppose that N wireless nodes have been randomly placed over a planar area. We are looking at a single episode of routing whereby some node c receives a packet addressed to some destination d and has to identify one of its neighbors to which the packet will be forwarded. The routing node c knows its own location, expressed as a pair of Cartesian coordinates, as well as the location of d and of all its neighbors. For simplicity, we assume that neighborhoods are determined based on a fixed propagation radius R. This means that the set of neighbors of c consists of those nodes whose coordinates fall into the circle with radius R centered at c. Given R and a density of node distribution, we consider the problem of finding the average number of hops required to traverse a given Euclidean distance in the network.

Note that this measure naturally translates into the expected end-to-end delay between a given pair of nodes; it can also be used to assess a *cost* of communication, e.g., understood as the required expense of energy. Another interesting application is the estimation of node location, e.g., by following the technique proposed in [5]. With that approach, some fixed *anchor* nodes flood the network, and then the unknown location of a node u is derived from the number of hops separating u from the anchors, assuming some (estimated) average size of a hop. Our analysis presented in this paper can greatly assist such estimations.

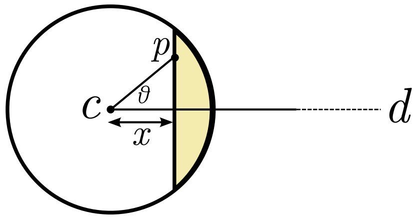

Fig. 1. Calculation of the Greedy CDF.

The distribution of the number of hops in a one-dimensional network, for a given transmission radius and node density, was studied in [6] as a prerequisite for estimating the number of broadcast cycles before hitting a communication void. However, that work did not consider the actual distance between nodes. Following up on [6], [7] gives an approximate distribution of the maximum Euclidean distance in a one-dimensional sensor network. The one-dimensional case is much simpler than 2-D where, in addition to the distance, the angle between the next hop node and the destination plays a paramount role. In [8], Zorzi *et al.* calculate the expected number of hops required by their protocol, dubbed GeRaF, also providing elegant upper and lower bounds on the number of hops for a given Euclidean distance separating the source form the destination.

In this paper, we focus on Greedy and Compass as the two fundamental generic schemes which, on reasonably dense and statistically balanced networks appear unbeatable in terms of average performance [4]. Thus, they should be viewed as prerequisites (or starting points) for all practical location based forwarding schemes.

III. Greedy

Let c be the forwarding node, as depicted in Figure 1, With Greedy, the search for a next hop neighbor of c is restricted to the angular sector of size π centered around the line connecting c to the destination d. This is because the next hop neighbor must reduce the packet's distance to the destination. Let p be the next hop node and θ be the angle formed by c, d, and p. Define $x = r\cos\theta$, where r is the normalized distance between c and p, i.e., $r \in [0, R]$ where R is the propagation radius. Let us assume that R is the same for all nodes and equal 1. The cumulative distribution function (CDF) of x and θ is defined as:

$$F(x,\theta) = P_r\{X \leq x \wedge \Theta \leq \theta\}$$

By applying the Bayes rule, we define:

$$G(x,\theta) = P_r\{\Theta \leq \theta | X \leq x\}\ ,\quad H(x) = P_r\{X \leq x\},$$

and obtain:

$$F(x,\theta) = G(x,\theta)\ H(x).$$

Note that, with respect to Figure 1, $H(x)$ gives the probability that there is no node in the shaded area. Thus we have:

$$H(x) = e^{-\rho A(x)}$$

where $A(x) = \cos^{-1} x - x\sqrt{1-x^2}$ is the size of the shaded area in Figure 1, and ρ expresses the density of the Poisson distribution of points in 2-D space. We assume that this density is sufficiently high for a next hop neighbor p to be always found (with the probability close to 1), i.e., the protocol does not have to cope with exceptions due to local maxima. Technically, this means that $e^{-\rho\pi/2} \sim 1$. To derive $G(x,\theta)$, we identify three cases:

1) $\theta \in [-\pi/2, -\cos^{-1} x]$:

$$G(x,\theta) = \frac{\pi/2 + \theta}{\pi - 2A(x)}$$

2) $\theta \in [-\cos^{-1} x, \cos^{-1} x]$:

$$G(x,\theta) = \frac{\pi/2 + x^2\tan\theta - A(x)}{\pi - 2A(x)}$$

3) $\theta \in [\cos^{-1} x, \pi/2]$:

$$G(x,\theta) = \frac{\pi/2 + \theta - 2A(x)}{\pi - 2A(x)}$$

By applying the Bayes rule and differentiating with respect to x and θ, we transform $F(x,\theta)$ into this density function:

$$f(x,\theta) = \begin{cases} \frac{B(x)e^{-\rho A(x)}}{\pi - 2A(x)} & ; \quad \theta \in [-\pi/2, -cos^{-1}x] \\ \frac{B(x)e^{-\rho A(x)}}{\pi - 2A(x)} x^2 sec^2\theta & ; \quad \theta \in [-cos^{-1}x, cos^{-1}x] \\ \frac{B(x)e^{-\rho A(x)}}{\pi - 2A(x)} & ; \quad \theta \in [cos^{-1}x, \pi/2] \end{cases} \tag{1}$$

where $B(x) = 2\rho\sqrt{1-x^2}$. Note that the marginal for x is $f(x) = B(x)e^{-\rho A(x)}$.

Figure 2 shows the shape of $G(x,\theta)$ and its corresponding density function $g(x,\theta)$. It additionally includes experimental points (the dashed lines) from Monte Carlo simulations to verify the analytical formulas.

IV. Compass

In the case of Compass, the next hop node p is decided based on the smallest angle from the straight line connecting c to d, as depicted in Figure 3. Note that in order for p to be selected, no other nodes should be present in the shaded are of Figure 3. It is also the case that in order to guarantee progress towards d, there must exist at least one node in the "forward" sector of angle π that is "facing" d. Thus, the possible angles are from $-\pi/2$ to $\pi/2$. Technically, the rules of Compass make sense for an arbitrary size of the forward sector $\theta_0 \leq \pi$, with the angle ranging from $-\theta_0/2$ to $\theta_0/2$ around the line connecting c to d. For the sake of brevity, the rest of the analysis considers the case of searching in the forward sector of size π, but the formulas can be easily generalized to any fixed θ_0. We return to consider various $\theta_0 < \pi$ in the next section (Figure 4).

The "forward sector" of size π can be split into two symmetric quadrants separated by the line connecting c and d. We will consider the analysis of the "positive" quadrant. Specifically, if $\theta \in [0, \pi/2]$, we are interested in finding:

$$F^{+}(x,\theta) = P_r\{\Theta \geq \theta \wedge X \leq x\} = P_r\{X \leq x | \Theta \geq \theta\} P_r\{\Theta \geq \theta\},$$

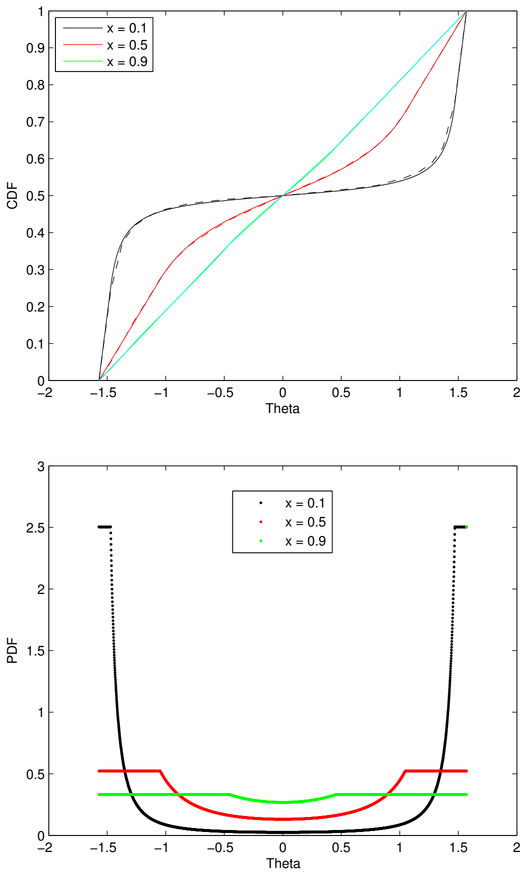

Fig. 2. The $G(x,\theta)$ and $g(x,\theta)$ of Greedy.

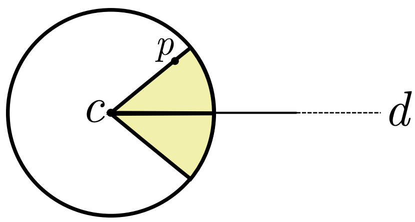

Fig. 3. Calculation of the Compass CDF.

and when $\theta \in [-\pi/2, 0]$, we are looking for the symmetric:
$F^-(x,\theta) = P_r\{\Theta \leq \theta \wedge X \leq x\} = P_r\{X \leq x | \Theta \leq \theta\} P_r\{\Theta \leq \theta\}$.

In the first case:

$$P_r\{\Theta \geq \theta\} = H^+(\theta) = 1 - e^{-\rho\theta},$$

and

$$P_r\{X \leq x | \Theta \geq \theta) = G^+(x,\theta) = \\ = \begin{cases} \frac{\pi/2 - (\cos^{-1} x - x\sqrt{1-x^2}) - x^2 \tan\theta}{\pi/2 - \theta} & ; \quad x \in [0, \cos\theta] \\ 1 & ; \quad x \in [\cos\theta, 1] \end{cases} \tag{2}$$

Then, we derive the respective density functions:

$$h^+(\theta) = \frac{d}{d\theta}(1 - e^{-\rho\theta}) = \rho e^{-\rho\theta}$$

and

$$g^+(x,\theta) = \begin{cases} \frac{2((1-\sqrt{1-x^2}) - x\tan\theta)}{\pi/2 - \theta} & ; \quad x \in [0, \cos\theta] \\ 0 & ; \quad x \in [\cos\theta, 1] \end{cases} \tag{3}$$

Now, the CDF for the "upper" quadrant takes this form:

$$F^+(x,\theta) = \begin{cases} \frac{e^{-\rho\theta}(\pi/2 - (\cos^{-1} x - x\sqrt{1-x^2}) - x^2 \tan\theta)}{(\pi/2 - \theta)} \\ e^{-\rho\theta} \end{cases} \tag{4}$$

with the density function:

$$f^+(x,\theta) = \begin{cases} \frac{2\rho e^{-\rho\theta}}{\pi/2 - \theta}(\sqrt{1-x^2} - x\tan\theta) & ; \quad x \in [0, \cos\theta] \\ 0 & ; \quad x \in [\cos\theta, 1] \end{cases} \tag{5}$$

The second case, $\theta \in [0, -\pi/2]$, being essentially symmetric, yields:

$$F^-(x,\theta) = \begin{cases} \frac{e^{\rho\theta}(\pi/2 - (\cos^{-1} x - x\sqrt{1-x^2}) + x^2 \tan\theta)}{(\pi/2 - \theta)} \\ e^{\rho\theta} \end{cases} \tag{6}$$

and

$$f^-(x,\theta) = \begin{cases} \frac{2\rho e^{\rho\theta}}{\pi/2 + \theta}(\sqrt{1-x^2} + x\tan\theta) & ; \quad x \in [0, \cos\theta] \\ 0 & ; \quad x \in [\cos\theta, 1] \end{cases} \tag{7}$$

V. Average Number of Hops

In the previous section we have determined the single hop PDF of $f(x,\theta)$ for the Greedy and Compass routing schemes. We will use it in this section to derive an approximation to the average number of hops from source to destination. The main approximating assumption is that each hop results in the same progress towards the destination, equal to the average distance covered in one hop. Indeed, if a node c is separated from the destination d by a distance D then after reaching the next hop, p, its remaining distance to the destination is D', and $D - 1 \leq D' < D$ (since the maximum hop distance is bounded by $R = 1$). The amount of progress accomplished in a single forwarding step, i.e., the *advancement* of the packet towards the destination is thus $L = D - D'$. $E[L]$ can then be calculated as:

$$E[L] = \int_{-\pi/2}^{\pi/2} \int_0^1 x f(x,\theta)\ dx d\theta$$

In the case of Greedy, this results in the following:

$$\begin{aligned} E[L] \quad &= \int_0^1 \int_{-\pi/2}^{-\cos^{-1} x} x \frac{(2\rho\sqrt{1-x^2})e^{-\rho A(x)}}{\pi - 2(\cos^{-1} x - x\sqrt{1-x^2})} d\theta dx \\ &+ \int_0^1 \int_{-\cos^{-1} x}^{\cos^{-1} x} x \frac{(x^2 \sec^2\theta)(2\rho\sqrt{1-x^2})e^{-\rho A(x)}}{\pi - 2(\cos^{-1} x - x\sqrt{1-x^2})} d\theta dx \\ &+ \int_0^1 \int_{\cos^{-1} x}^{\pi/2} x \frac{(2\rho\sqrt{1-x^2})e^{-\rho A(x)}}{\pi - 2(\cos^{-1} x - x\sqrt{1-x^2})} d\theta dx \end{aligned} \tag{8}$$

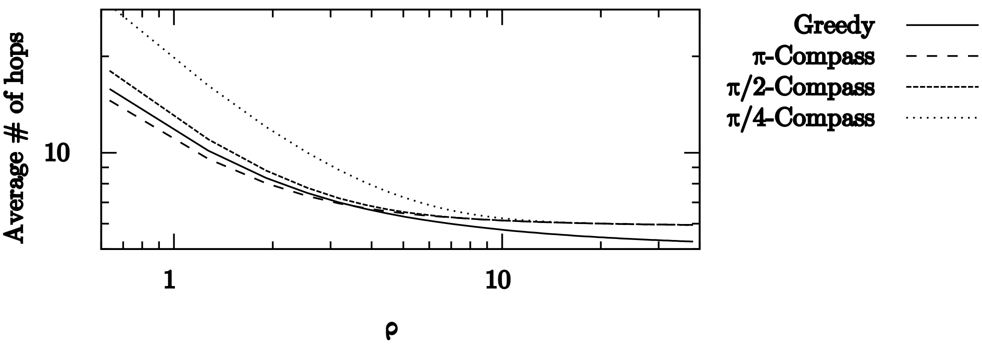

Fig. 4. Average number of hops for source to destination distance of 5 units.

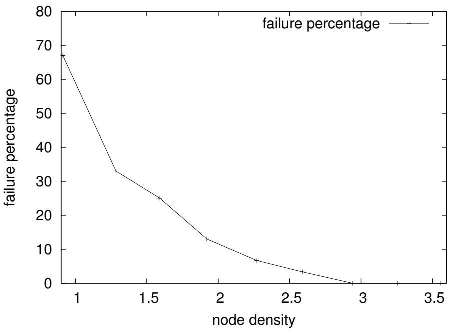

Fig. 5. Percentage of route failures.

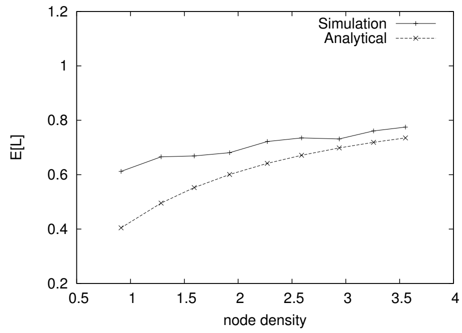

Fig. 6. Model versus simulation: Greedy.

and for Compass we get:

$$\begin{aligned} E[L] &= \int_{-\pi/2}^{0}\int_{0}^{\cos\theta} \frac{2x\rho e^{\rho\theta}(\sqrt{1-x^2}+x\tan\theta)}{(\pi/2+\theta)}\ dxd\theta \\ &+ \int_{0}^{\pi/2}\int_{0}^{\cos\theta} \frac{2x\rho e^{-\rho\theta}(\sqrt{1-x^2}-x\tan\theta)}{(\pi/2-\theta)}\ dxd\theta \qquad (9) \end{aligned}$$

Figure 4 illustrates the impact of node density ρ on the average number of hops $E[h]$ using the approximation $E[h] = D/E[L]$. The results shown are for a source to origin distance equal to 5 distance units (essentially five times the radius of each node's transmission). Clearly the asymptotic average number of hops comes close to 5 for high node density. However, the asymptotic behavior of the Compass schemes is towards a slightly higher number of hops than Greedy, even as node densities become extremely high. Thus, Compass, even if restricted to a small sector (as demonstrated by the $\pi/2$ and $\pi/4$ variants) is systematically penalized. It is somewhat surprising that at low densities Greedy and Compass (π–Compass) behave equally, and the impression may be given that π–Compass could even outperform Greedy. Recall, however, that at small densities the schemes may exhibit route failures as it is no longer true that $e^{-\rho\pi/2} \sim 1$.

Figure 5 shows the results of a simulation experiment carried out to determine the percentage of route failures in random networks of different densities with the source and destination separated by 5 propagation radii. As we can see, for ρ less than about 1.7, the percentage of failures (which is clearly the same for both schemes) exceeds 20%, which means that for that range of densities the analytical model will break down. Consequently, the indications of Figure 4 for low values of ρ cannot be considered reliable.

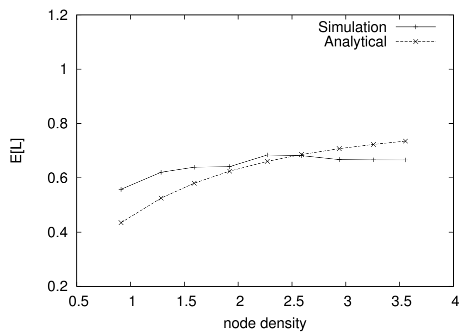

Fig. 7. Model versus simulation: Compass.

In Figures 6 and 7 we compare the number of hops predicted by our model with that observed in a simulation experiment carried out for networks of various density ρ. In both cases, the discrepancy between the model and the simulation becomes considerably more pronounced for small ρ, where the assumption about the infallibility of the routing rules ceases to hold with a satisfying probability.

VI. CONCLUSIONS

We have shown an analytical model for estimating the number of hops (and thus the path length) in reasonably dense random networks operating under two generic location-based forwarding schemes: Greedy and Compass. The model assumes that the network density makes it always possible to find a next-hop node according to the simple principle of following a purely local optimum. While realistic incarnations of Greedy and Compass must include supplementary rules allowing the schemes to cope with blind alleys and loops, the performance of a sufficiently dense network will be statistically dominated by the simple paradigm. As the efficiency of that paradigm is practically unbeatable, any enhancements of Greedy and Compass will tend to avoid spoiling it as much as possible. Consequently, our model will be applicable to such networks with a good accuracy.

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