On Constructing Minimum-Energy Path-Preserving Graphs for Ad-Hoc Wireless Networks

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***Abstract*— We consider the power control problem in ad-hoc wireless networks. Local decisions regarding the transmission power level induce a subgraph of the maximum powered graph G_{max} in which edges represent direct reachability at maximum power. We propose a new algorithm for constructing minimum-energy path-preserving subgraphs of G_{max}. Its superiority over previous solutions demonstrates once again that strict protocol layering in wireless networks tends to be detrimental to performance.**

***keywords:* Ad-hoc Network, Topology Control, Connected Graph, Medium Access Control.**

I. INTRODUCTION

Consider an n-node multi-hop ad-hoc wireless network on a two-dimensional plane. Each node is capable of adjusting its transmission power up to P_{max}. Such a network can be modeled as a graph $G = (V, E)$,

$$E = \{\langle x, y\rangle \mid \langle x, y\rangle \in V \times V \wedge d(x, y) \leq R_{max}\}\ ,$$

where $d(x, y)$ is the distance between nodes x and y and R_{max} is the maximum distance reachable by a transmission at the maximum power P_{max}. This (symmetric) model assumes that the minimum power needed to reach a node depends solely on the distance to the node. The graph G is called the *maximum powered network*.

The local choices regarding the transmission power at individual nodes collectively shape a subgraph of G. Its average degree has a significant impact on the performance of the routing layer, e.g., it may cause a *broadcast storm* problem in a dense network [17]. The issue of selecting the optimum transmission power formulated in this context was first tackled by Roduplu et al. [14].

We say that a graph $G' \subseteq G$ is a *minimum-energy path preserving graph* [6], if for any pair of nodes (u, v), at least one of the minimum energy paths between u and v in G also belongs to G'. Typically, many minimum-energy path preserving graphs can be formed from the original graph G. It has been shown that the smallest of such subgraphs is the graph $G_{min} = (V, E_{min})$, where $(u, v) \in E_{min}$ iff there is no path of length greater than 1 from u to v that costs less energy than required for a direct transmission between u and v.

Let $G_i = (V, E_i)$ be a subgraph of $G = (V, E)$ such that $(u, v) \in E_i$ iff $(u, v) \in E$ and there is no path of length i that requires less energy than the direct one-hop transmission between u and v. Then G_{min} can be formally defined as follows:

$$G_{min} = \bigcap_{i=2}^{n-1} G_i$$

It is easy to see that any subgraph G' of G has the *minimum-energy property* iff $G' \supseteq G_{min}$. Thereby, each of $G_i \supseteq G_{min}$, for any $i = 2, 3, \ldots, n-1$ is a *minimum-energy path preserving graph*.

Distributed construction of G_{min} contradicts its own goals, because it requires communicating with distant nodes using high power. On the other hand, graphs G_i can be built based on local information at relaxed power requirements. An algorithm for constructing G_2, was presented in [6]. It works reasonably well in dense networks but its performance degrades considerably when the network density drops.

In this paper, we show how to construct G_2 efficiently in sparse and moderately dense networks with some assistance of the Medium Access Control (MAC) layer. The proposed modifications affect the backoff procedure of 802.11b and are somewhat reminiscent of [2]. In our own previous work [11], we proposed another modification to the collision avoidance mechanism of 802.11b aimed at improving the reliability of multicast transmissions.

II. RELATED WORK

There have been three general approaches to tackling power control issues in wireless networks. The first class of proposed solutions deals with the MAC layer. Monks *et al.* [9] propose a modification to the IEEE 802.11 RTS-CTS handshake whereby a node overhearing a CTS packet uses the received power to estimate its distance to the sender. Also, Sing *et al.* [15] propose techniques for powering-off the transceivers when they are not active.

The second approach is the so-called *power-aware routing* [16]. It can be combined with the third approach which separates routing from topology control. Ramanathan et al. [12] describe two centralized heuristic algorithms to minimize the maximum transmission power while trying to maintain connectivity or bi-connectivity, called Local Information No topology (LINT) and Local Information Link-State

Topology (LILT). Cone-Based Topology Control (CBTC), proposed by Li *et al.* [7] generates a graph structure similar to the one proposed by Yao [20]. One issue with that algorithm is the determination of the suitable initial power level and the increment of power level at each step. The choice of these two parameters may have significant impact on the number of overhead messages.

Narayanaswamy *et al.* [10] propose a power control protocol named COMPOW. Their objective is to choose the smallest common power level for each node that 1) maintains connectivity, 2) maximizes traffic carrying capacity, 3) reduces contention in the MAC layer, and 4) requires low power. The drawback of their approach is its assumption of homogeneous distribution of nodes. CLUSTERPOW [5] was designed to overcome the shortcomings of COMPOW by introducing node clusters. However, it still suffers from a significant message overhead.

N. Li *et al.* [8] propose a distributed topology control algorithm (LMST) based on constructing minimum spanning trees. Their algorithm achieves 3 goals: 1) connectivity, 2) bounded node degree (≤ 6), and 3) providing bi-directional links. One problem with LMST is that the final topology does not preserve the minimum-energy paths between nodes.

Rodoplu *et al.* [14] introduce the notion of *relay region* based on a specific power model. Their algorithm was later modified by Li *et al.* [6] to remove some unnecessary edges. Our work closely relates to these two studies and a discussion of the respective algorithms is given in the next section.

Among other work in this area, Wattenhofer *et al.* [19] propose a two-phased algorithm which consists of creating a variation of the Yao graph followed by a Gabriel Graph. Huang et al. [4] propose a topology control algorithm taking advantage of directional antennas.

III. Minimum-Energy Path Preserving Graphs

A. Power model

We assume the generic, two-ray, channel path loss model [13]. To send a packet from node x to node y, separated by distance $d(x,y)$, the minimum transmission power is $P_{trans}(x,y) = t \times d^{\alpha}(x,y)$, where $\alpha \geq 2$ is the *path loss factor* and t is a constant. Signal reception is assumed to cost a fixed amount of power denoted by r. Thus, the total power expended on one-hop transmission between x and y becomes: $P_{total}(x,y) = t \times d^{\alpha}(x,y) + r$. The model assumes that each node is aware of its own position with a reasonable accuracy, e.g., via a GPS device.

B. Previous approach to constructing G_2

The algorithm presented in [14] is based on the notion of *relay-region*. Given a node u and another node v within u's communication range (at P_{max}), the relay region of node v as perceived by u, $R_{u \to v}$, is the collection of points such that relaying through v to any point in $R_{u \to v}$ takes less energy than a direct transmission to that point. To construct G_2, suppose that u is the starting node of a path. If, as perceived by u, w falls in the relay region of v, then w will not be included in the *neighborset* of u (u will not transmit directly to w). G_2 is constructed by connecting each node with only those nodes that are included in its neighborset. The attractiveness of this approach depends on how efficiently nodes collect the position information of neighbors. One trivial (and power-expensive) way is to periodically broadcast a *neighbor discovery message* (NDM) at P_{max}, to which all reachable nodes will respond with position information.

Given $R_{u \to v}$, the complement region, denoted by $R^c_{u \to v}$, is the set of points for which it is not power-efficient for u to use node v as a relay. Let $N(u)$ be the set of nodes that do not fall in the relay region of any other node in u's neighborhood. Then, $\bigcap_{k \in N(u)} R^c_{u \to k}$ is the set of points where u should transmit directly without using any relay. On the other hand, the direct transmission range of u is limited by P_{max}—the maximum transmission power. Let $F(u, P_{max})$ denote the circular region with radius R_{max} centered at u and describing its transmission range. The *enclosure* of node u is defined as the set of points: $\epsilon_u = \bigcap_{k \in N(u)} R^c_{u \to k} \bigcap F(u, P_{max})$.

The observation made in [6], that a node need not search for neighbors beyond the enclosure, lead to a power saving algorithm for constructing G_2 dubbed RNSA, for *Reduced Neighbor Search Algorithm*. Instead of broadcasting the NDM at the maximum power, u starts with some initial power, P_0. After collecting responses from the neighborhood, if the enclosure has been found, then there is no need to search any further. Otherwise u will re-broadcast the NDM at an increased power level and try again. This process will continue until u either finds the enclosure or reaches P_{max}.

C. Problems with RNSA

While the algorithm works fine when the network is dense, in a moderately populated network it tends to exhibit poor performance. This is because the enclosure of a node u can be formed in one of two possible ways: (i) when this condition holds: $\bigcap_{k \in N(u)} R^c_{u \to k} \subseteq F(u, P_{max})$, the enclosure is determined solely by the nodes in $N(u)$; (ii) the transmission range of u is a limiting factor, i.e., $\bigcap_{k \in N(u)} R^c_{u \to k} \neq \bigcap_{k \in N(u)} R^c_{u \to k} \bigcap F(u, P_{max})$. If a node has an enclosure of type (i), called an *enclosure by neighbors*, then, in principle, it need not transmit at P_{max} to find that enclosure. For such nodes, RNSA is useful and may result in power savings compared to the naive scheme. On the other hand, a node having an enclosure of type (ii), called an *enclosure by maximum boundary*, will ultimately need to search with P_{max}. For such a node, RNSA performs worse than the naive scheme as it runs through a number of futile iterations before reaching P_{max}.

Figure 1 shows some statistics relating the observed percentage of nodes with enclosures by neighbors to the network density. The density of the network is this experiment was determined by the total number of nodes, which were distributed uniformly in a fixed square region of $670m \times 670m$. This picture clearly suggests that RNSA will not perform well for sparse networks, where many nodes have to transmit at P_{max} to find their enclosures.

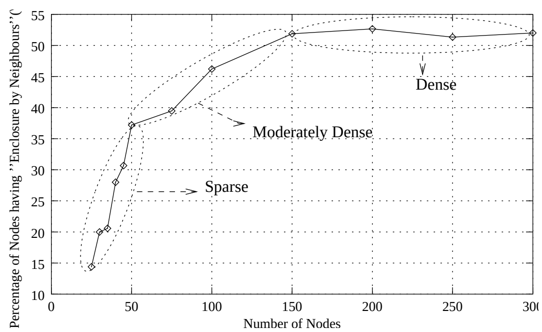
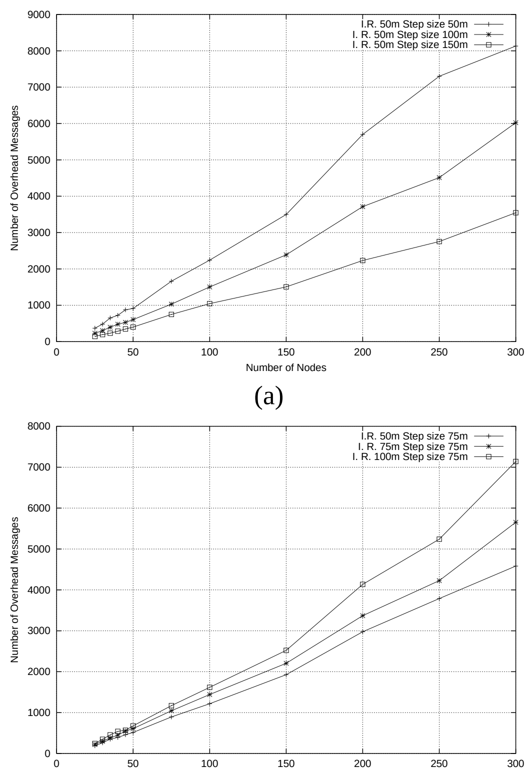


Fig. 1. Percentage of nodes with enclosures by neighbors

The second problem with RNSA is the lack of guidelines regarding the selection of the initial power and the increment. Figure 2, showing the relationship between those parameters and the resulting message overhead of RNSA, demonstrates that their choice is not irrelevant.



(b)

Fig. 2. Comparing number of overhead messages by varying (a) Step size, (b) Initial communication range

In part (a), the initial transmission range is the same for all three curves ($50m$), but the increments are different: $50m$, $100m$, and $150m$. Especially for dense networks, where RNSA is most useful, the differences are considerable and exceed 50%. In part (b), the step size is fixed at $75m$ while the initial transmission range varies between $50m$ and $100m$. As the observed susceptibility of the algorithm's performance to the parameters is rather high, one can expect that their optimum setting is also highly sensitive to the dynamic characteristics of ad-hoc networks.

IV. Constructing G_2 for Non-Dense Networks

A. Cover Region and Cover Set

Consider a pair of nodes (s, f), such that f is reachable by s at P_{max}. Envision the the set of all points that can host relays between s and f, such that it would be more power efficient for s to use a relay rather than send directly to f. Owing to the symmetry of the propagation model, exactly the same set is defined by considering f as the starting point. We call it the *cover region* of s and f and denote by $C_{(s,f)}$. The collection of all nodes falling into the cover region of s and f is called the cover set of s and f. Formally the cover region and cover set, are described by the following definition.

Definition 1: The cover region $C_{(s,f)}$ of a pair of nodes (s, f), where f is reachable from s, is defined as:

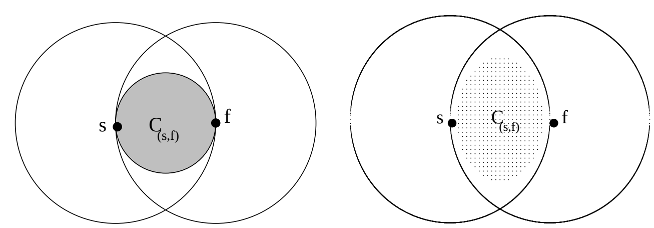
$$C_{(s,f)} = \{<x,y> \mid td^{\alpha}(s,<x,y>) + td^{\alpha}(<x,y>,f) + r \leq td^{\alpha}(s,f)\} \qquad \text{where}\, \alpha \geq 2$$

In the above equation, $d(s,<x,y>)$ denotes the distance between node s, and a hypothetical node located at $<x,y>$. The cover set of the same pair (s, f) is $\xi_{(s,f)} = \{v \mid v \in V \wedge Loc(v) \in C_{(s,f)}\}$. Figure 3 shows two examples of cover regions, with the path loss exponent $\alpha = 2$ and $r = 0mW$, and $\alpha = 4$, $r = 20mW$.

Lemma 1 : (a) For any $c \in \xi_{(s,f)}$, $d_{sc} < d_{sf}$, (b) If $c \in \xi_{(s,f)}$ then $f \notin \xi_{(s,c)}$.

Proof : (a) If $c \in \xi_{(s,f)}$ then from the definition 1 it follows that, $d_{sc}^{\alpha} + d_{cf}^{\alpha} + r/t \leq d_{sf}^{\alpha}$. Now for $r > 0$ and $\alpha \geq 1$, $d_{sf} > d_{sc}$.

(b) If $c \in \xi_{(s,f)}$, then from (a) $d_{sf} > d_{sc}$. Now suppose that also $f \in \xi_{(s,c)}$ then again from (a), $d_{sc} > d_{sf}$ which is a contradiction.



(a) (b)

Fig. 3. Cover regions: (a) $\alpha = 2$, $r = 0mW$ (b) $\alpha = 4$, $r = 20mW$

B. Constructing G_2

The operation is described from the viewpoint of one node s. In contrast to RNSA, s broadcasts a single neighbor discovery message (NDM) at P_{max}. For now, let us assume that all nodes receiving the NDM from s send back a reply. The reduced overhead in our algorithm will result from reducing the number of replies.

Whenever s receives a reply to its NDM from a node v, it performs the algorithm listed in Figure 4. Its purpose is to update the configuration of the cover sets $C_{(s,w)}$ and discover nodes in its neighbors A_s (all sets are initialized to empty). At the end, when s has received all replies, the configuration of cover sets is complete.

$$
\begin{array}{l}
\textbf{updateCoverRegion}(s, v) \\
begin \\
\quad for\ each\ w \in A_s \\
\quad\quad if\ Loc(v) \in C_{(s,w)}\ then \\
\quad\quad\quad \xi_{(s,w)} = \xi_{(s,w)} \cup \{v\}; \\
\quad\quad else\ if\ Loc(w) \in C_{(s,v)}\ then \\
\quad\quad\quad \xi_{(s,v)} = \xi_{(s,v)} \cup \{w\}; \\
\quad A_s = A_s \cup \{v\}; \\
end
\end{array}
$$

Fig. 4. Algorithm for building cover sets

Having determined the cover regions of all its neighbors, s is in position to identify the members of its neighborset. If $\xi_{(s,v)} \neq \emptyset$ for some v, it means that there is at least one node $w \in \xi_{(s,v)}$ that can act as a power-efficient relay between s and v. On the other hand, a node v that has no nonempty cover set with s, but belongs to the neighborhood of s (is present in A_s) necessarily has no power-efficient relays and thus belongs to the neighborset of s. The loop listed in Figure 5 completes the algorithm by generating the neighborset of s denoted by $\aleph_s$.

$$
\begin{array}{l}
\textbf{neighbor}(s) \\
begin \\
\quad \aleph_s = \emptyset \\
\quad for\ each\ v \in A_s \\
\quad\quad if\ \xi_{(s,v)} = \emptyset\ then \\
\quad\quad\quad \aleph_s = \aleph_s \cup \{v\}; \\
end
\end{array}
$$

Fig. 5. Generating the neighborset of s

C. Reducing the Number of Reply Messages

Consider a simple scenario where s can reach only two nodes v and w within the radius of maximum transmission range, such that $v \in \xi_{(s,w)}$. From Lemma 1(b), $w \notin \xi_{(s,v)}$ and $\aleph_s = \{v\}$. When s broadcasts its NDM, both v and w are supposed to send back a reply. One of the messages will be transmitted and received by s before the other. Note that if v wins, the message sent by w will not affect the outcome of the algorithm as w is covered by v and it should not be included in $\aleph_u$. On the other hand, if w wins, the algorithm will first add w to A_u and then, after receiving the second message from v, add v to $\xi_{(s,w)}$. Note that if w overheard the reply of v, then w could refrain from sending its reply to s.

To give precedence to those replies that are likely to be relevant, we need some control over the contention resolution mechanism in the MAC layer. With IEEE 802.11b, a node willing to transmit a packet under contention has to wait for a certain number of idle slots chosen at random from $[0, cw-1]$, where cw is the *contention window*. To influence the order of transmission in a way that would be compatible with our sense of relevance, we propose to bias the random distribution of the slot selection process. This bias should not eliminate the randomness completely–to avoid permanent lockouts in the unavoidable situations when two or more nodes consider themselves equally relevant based on their local perception of the environment.

The idea is to make the expected waiting time a function of the distance from the node that sent the NDM. According to Lemma 1(a), if a node v is in the cover set of node w, then d_{sv} must be less then d_{sw}. Let $F(s, P_{max})$ represent the circular region of radius R_{max} reachable by s at its maximum transmission power. We divide $F(s, P_{max})$ into n equally-sized partitions. A node v is said to fall into partition i, $1 \leq i \leq n$ iff $R_{max} \times (i-1) < d_{sv} \leq R_{max} \times i$. A node in partition i will choose a random number prescribed by the following equation:

$$R = (i-1) \times \frac{cw}{2^{\lceil log_2 n \rceil}} + \lfloor U(0,1) \times \frac{cw}{2^{\lceil log_2 n \rceil}} \rfloor \ .$$

Example: Let $n = 2$. The transmission range of s is divided into two partitions. Nodes whose distance from s is less than $R_{max}/2$ are assigned to partition number 1, and all the remaining nodes fall into partition 2. If the current contention window size $cw = 32$, then nodes in partition 1 will choose a random number between 0 to 15 and nodes in partition 2 will choose a random number between 16 to 31.

V. Experimental Results

Our simulation model was based on *ns*-2 [1] with wireless extensions. The distributed coordination function (DCF) of the IEEE standard 802.11 [3] was used as the MAC layer. The radio model characteristics were similar to Lucent's WaveLAN [18].

Initially, we deployed $25-50$ nodes over a flat square area of $670m \times 670m$, with $R_{max} = 250$m. We ran experiments to see the effect of the varying partition size on the performance of our algorithm, specifically the ability of the biased backoff function to assist us in prioritizing the reply messages. The performance measure of interest was the *Saving Ratio* defined as follows:

$$\text{Saving Ratio} = \frac{N_{cancel}}{N_{sent} + N_{cancel}} \times 100(\%) \ ,$$

where N_{sent} is the total number of reply messages sent for each NDM requests, and N_{cancel} is the number of messages that have been eliminated as redundant. The savings (Figure 6) appear to be considerably higher for sparser networks. Finer partitions also tend to exhibit slightly better performance.

Figure 7 compares the performance of our algorithm with RNSA. The *Saving Index* is defined as follows:

$$\text{Saving Index} = \frac{N_{RNSA} - N}{N_{RNSA}} \times 100(\%) \ ,$$

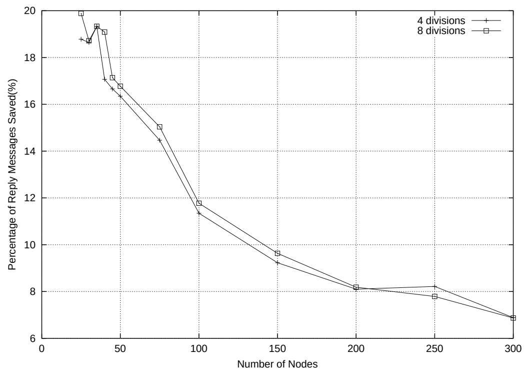


Fig. 6. Savings versus number of nodes

where N is the total number of reply messages needed by our algorithm to construct G_2, and N_{RNSA} is the corresponding number of messages in RNSA.

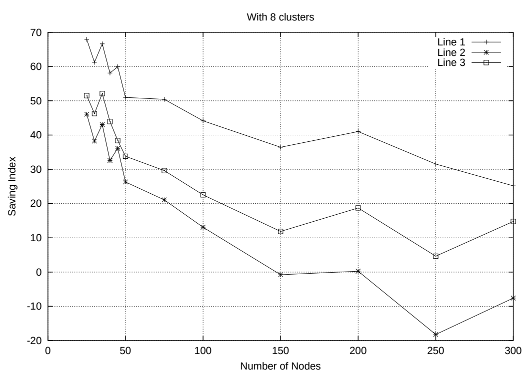


Fig. 7. Comparison with RNSA

The three different curves correspond to the different settings of parameters for RNSA (see Table I). For all curves, the number of partitions used by our algorithm was 8.

TABLE I
SIMULATION PARAMETERS

Curve Name	Initial Range (m)	Step Size (m)
Line 1	50	50
Line 2	75	75
Line 3	100	75

For sparse networks, the *Saving Index* is very high and drops with the increasing density of nodes. In particular, $25 - 65\%$ of reply messages were eliminated with our approach for less then 50 nodes, and $15 - 45\%$ savings were observed for networks between 50 and 100 nodes.

VI. Conclusion

We have presented a MAC-assisted algorithm for constructing minimum energy path preserving graphs in ad-hoc networks. Our studies have demonstrated the superiority of the new algorithm over the previous solution for networks with moderate and low density of nodes. Our exercise shows that the issue of power control calls for the collaboration of all layers, and keeping some layers closed may significantly impair the flexibility of the whole protocol stack.

Acknowledgment

The authors would like to thank Joseph Halpern and Li Li for a valuable clarification of their algorithm [6].

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