Adaptive Probabilistic Medium Access in MPR-Capable Ad-hoc Wireless Networks

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***Abstract*—Medium access in ad-hoc wireless networks must be performed in a distributed fashion due to lack of coordination between nodes. Specifically, when nodes are capable of receiving more than one transmission simultaneously, the design of distributed medium-access mechanisms that efficiently exploit the receiver's capability becomes more challenging.**

Adaptive probabilistic medium access for ad-hoc wireless networks is proposed in this paper. Nodes with data packets to transmit perform an *announcement process* in order to inform other nodes of their intended traffic. The acquired information through this process about other potential transmitters in the vicinity is then used by the nodes to choose a *transmission probability* with which they transmit their data packets. The performance of a multi-packet reception capable ad-hoc wireless network under the proposed protocol is analyzed and evaluated numerically and via simulations, and compared with Aloha-type random access.

I. Introduction

After successful realizations of infrastructure wireless networks in the fields of cellular communications and local area networking, infrastructure-less networking is going to play a key role in the next generation of wireless networks. Applications of such networks, also known as ad-hoc wireless networks, include networking wireless sensors in industrial environments, surveillance, disaster recovery, and distributed computing. The fundamental differences between these networks and their traditional infrastructure counterparts raise new challenges. Examples are *distributed* algorithms for routing, medium access, quality of service, and security [1], low-power implementations, and information theoretic evaluations of such networks [2].

Multi-packet reception (MPR), powered by new technologies such as code-division multiple-access, multiple-antenna arrays, and space-time coding, is the ability to receive multiple communications packets simultaneously. The ability of receiving concurrent transmissions deviates from the traditional *collision* model and, hence, makes motivation towards designing medium access mechanisms to be able to exploit this feature.

Due to lack of central coordination, medium access in ad-hoc wireless networks must be controlled by distributed mechanisms. Specifically, when nodes are capable of receiving concurrent transmissions, they need distributed protocols to exploit the capability optimally in the the sense of using available bandwidth while avoiding excessive concurrent transmissions.

To approach this goal, we propose *adaptive probabilistic medium access* through which a potential transmitter acquires an estimate of the number of on-going transmissions in its vicinity and, then, transmits its packet with a probability adapted to the current capacity of the receiver. In this method, having more potential transmitters in an area leads to lower *transmission probabilities*. Finding the optimum transmission probability leads to an optimization problem with regard to overall network performance.

In this paper, the performance of an MPR-capable ad-hoc wireless network under the proposed protocol is studied. The network performance is analyzed and evaluated numerically and via simulations, and compared with Aloha-type random access. Hence, the scope of this paper is not to propose a detailed medium-access control (MAC) protocol, but to introduce a feasible medium-access mechanism followed by an analytical framework.

The rest of the paper is organized as follows. In Section II, the related literature is reviewed. In Section III, the system model is introduced and the proposed protocol is described. The network performance is analyzed in Section IV. Numerical evaluations and simulation results are provided in Section V. Finally, Section VI concludes the paper.

II. Related Literature

MPR-capable cellular wireless networks have been studied and several MAC protocols have been proposed in the literature [3]–[6]. The protocol proposed in [3] utilizes explicit reservations. In this method, time is divided into slots, each composed of two sub-slots: reservation and data transmission. Nodes with packets to transmit enter the reservation phase that *deterministically* specifies the winners. Similarly, protocols proposed in [4] and [5] require central controllers at the base stations.

A predictive protocol utilizing a finite-size buffer was proposed in [7]. The performance of the protocol and the effect of the buffer was analyzed using the theory of discrete-time Markov chains. The analysis, however, is computationally complex and was offered as a complement to simulations. In [6], a multi-group priority queuing MAC protocol for cellular wireless networks was proposed. The protocol performs a user classification based on the activity history and applies a *deterministic* control policy over users' transmissions.

Towards design and analysis of ad-hoc medium access protocols, Nagaraj *et al.* [8] analyzed the performance of Aloha-type random access in a fully-uncoordinated network. It was shown that random access reaches the optimal throughput asymptotically as the MPR capability goes to infinity. In [9], the local and multihop throughput of slotted Aloha in MPR-capable ad-hoc wireless networks with a number of receiver models were analyzed. The majority of the analysis was performed numerically and verified via simulations.

Finally, a distributed probabilistic access protocol that adjusts transmission probabilities based on the history of successes and failures was proposed in [10]. Each transmitting node maintains a status based on feedbacks from its intended receiver and selects the transmission probabilities accordingly.

III. Adaptive Probabilistic Medium Access

A. System Model

The system is a network composed of a number of nodes randomly distributed in a two-dimensional area. The node distribution over the network area follows a Poisson point process. Data packets are assumed constant-length and each node has a *slotted* interpretation of the time *independent* of other nodes.

For data communications, each node, if not transmitting, is able to receive up to K packets simultaneously. That is, the MPR capability of receivers is K. The event that a receiver is unable to receive packets due to being exposed to more than K concurrent transmissions is called an interference *outage*. In addition to the data channel, there is a *logical* announcement channel realized, for example, as proposed in [11]. Both the data and announcement channels are shared by all users.

B. Protocol Description

Adaptive probabilistic medium access can be summarized as follows: Nodes with data packets to transmit perform an *announcement process* in order to inform other nodes of their intended traffic. The acquired information through this process about other potential transmitters is then used by the nodes to choose a transmission probability with which they transmit their data packets.

Definition 1: A node is called an *active node* if it has a data packet to transmit.

Definition 2: For a specific set of node locations, a node A is a *neighbor* of another node B if a packet from A can be received by B. We assume that being neighbors is reciprocal. The *neighborhood* of a node is the receive footprint in the sense above.

Through the announcement channel, each active node informs its neighborhood of data about to be transmitted. The announcement channel is a lightly-loaded Aloha-type random access channel in order to provide a low-error communications medium without imposing extra coordination. A realization of such a channel that utilizes the MPR capability is the *header channel* mechanism in RP-CDMA [11].

In the rest of this paper, we assume error-free communications over the announcement channel due to its light traffic load. Hence, an active node can acquire the approximate number of other active nodes in its vicinity by listening to the announcement channel. Then, it chooses a transmission probability p with which to transmit its data packet at the beginning of its time slot.

In general, p can be calculated based on various network parameters, e.g. active nodes' distribution, quality-of-service and fairness requirements as well as power consumption considerations. In this paper, we consider the case where p is calculated as a function of the MPR capability K and the number of active nodes M to optimize the overall system performance. A first candidate for p is

$$p = \begin{cases} 1 & M \le K \\ K/M & \text{otherwise.} \end{cases} \quad (1)$$

IV. Performance Evaluation

To study the effect of p, we calculate performance metrics of a single-hop network. Let M be the number of active nodes in a receiver's neighborhood. We assume that active nodes in the vicinity know M. For a spatially-distributed network, this knowledge may be imprecise due to hidden and/or exposed nodes. In the next section, however, we will see that these effects are negligible when the MPR capability K becomes large.

A. Throughput

When M active nodes choose a common transmission probability p, the throughput becomes

$$\begin{aligned} R(p;K,M) &= \mathrm{E}(N_{\mathrm{TX}}^{\mathrm{succ}}) \\ &= \sum_{n=1}^{K} n \Pr(N_{\mathrm{TX}} = n) \\ &= \sum_{n=1}^{K} n \binom{M}{n} p^n (1-p)^{(M-n)}, \end{aligned} \quad (2)$$

where N_{TX} denotes the binomially-distributed total number of transmitted packets, and $N_{\mathrm{TX}}^{\mathrm{succ}}$ is the number of successfully-received packets obtained by

$$N_{\mathrm{TX}}^{\mathrm{succ}} = N_{\mathrm{TX}} \mathbf{1}_{(N_{\mathrm{TX}} \le K)}.$$

B. Transmission Power efficiency

An important performance metric for mobile devices is the transmission power efficiency, i.e. the ratio of the transmission power of successfully-received packets over the total transmission power. Similar to (2), the transmission power efficiency follows

$$\begin{aligned} \eta(p;K,M) &= \frac{\mathrm{E}(N_{\mathrm{TX}}^{\mathrm{succ}})\mathcal{E}_{\mathrm{TX}}}{\mathrm{E}(N_{\mathrm{TX}})\mathcal{E}_{\mathrm{TX}}} \\ &= \frac{R(p;K,M)}{Mp}, \end{aligned} \quad (3)$$

where $\mathcal{E}_{\mathrm{TX}}$ is the transmission energy of a packet.

C. Average Delay

We calculate the average delay that a packet experiences in the MAC layer with the buffer size 1. When a packet arrives in the MAC buffer, depending on the local traffic load, the packet either may be transmitted immediately or may have to wait for a number of time slots. Hence, the total delay experienced by the packet, including the transmission delay, is

$$d = T_{\mathrm{slot}}\,(w+1),$$

where T_{slot} is the slot duration and w is the number of slots that the packet waits. Assuming a stationary traffic, the average delay is

$$\mathrm{E}(d) = T_{\mathrm{slot}}\big(\mathrm{E}(w)+1\big). \tag{4}$$

In general, the average delay is a function of the outage probability when the MAC layer supports retransmission upon outage, *and* retransmissions are frequent. The probability of successful reception follows

$$P_{\mathrm{TX}}^{\mathrm{succ}}(p;K,M) = p\left(1 - P_{\mathrm{out}}(p;K-1,M-1)\right).$$

Assuming a constant p due to stationarity, we obtain

$$\begin{aligned}\mathrm{E}(w) &= \sum_{i=1}^{\infty} i P_{\mathrm{TX}}^{\mathrm{succ}}(p;K,M)\left(1 - P_{\mathrm{TX}}^{\mathrm{succ}}(p;K,M)\right)^{i}\\ &= \frac{1 - P_{\mathrm{TX}}^{\mathrm{succ}}(p;K,M)}{P_{\mathrm{TX}}^{\mathrm{succ}}(p;K,M)},\end{aligned}$$

and, thus,

$$\mathrm{E}(d) = \frac{T_{\mathrm{slot}}}{p\,(1 - P_{\mathrm{out}}(p;K-1,M-1))}, \tag{5}$$

where $P_{\mathrm{out}}(p;K,M)$ is the outage probability. According to the model, an outage happens when the number of concurrent transmissions exceeds K. Hence,

$$\begin{aligned}P_{\mathrm{out}}(p;K,M) &= \Pr(N_{\mathrm{TX}} > K)\\ &= 1 - \Pr(N_{\mathrm{TX}} \le K)\\ &= 1 - \sum_{n=0}^{K}\binom{M}{n}p^{n}(1-p)^{(M-n)}.\end{aligned} \tag{6}$$

When the MAC layer does not have a retransmission policy or when outages are rare, the delay takes a simple exponential form with the mean T_{slot}/p.

D. Performance in Spatially-Distributed Networks

As mentioned at the beginning of this section, the model used for calculations was a fully-interconnected network, i.e. a network in which every pair of nodes are neighbors. When nodes are spatially distributed, a difference between the number of active nodes hidden from transmitter and the number of active nodes exposed to the transmitter causes inaccuracies of the estimate of M. In the following, however, it is shown that this effect can be neglected.

Lemma 1: In a Poisson point process with density ρ, the probability of having a difference n between the numbers of points in two disjoint equi-spacious zones is

$$\exp(-2\rho A) I_{|n|}(2\rho A),$$

where A is the area of each of the two zones, and $I_{\alpha}(.)$ is the modified Bessel function of the first kind of order α.

Proof: See Appendix. ■

Theorem 2: Consider a network whose nodes are spatially distributed according to a Poisson point process. Assume that at a specific time slot, each node, independent of other nodes, is active with probability α which is identical for all nodes. Also assume that the neighborhood areas of all nodes are identical. Then, the probability that a transmitter *significantly* miscounts the number of receiver's active neighbors asymptotically goes to zero as $K \to \infty$.

Proof: See Appendix. ■

In the next section, we will see that the degradation of performance due to imprecise information is negligible for an MPR capability of as little as $K = 10$.

V. Numerical and Simulation Results

Choosing the transmission probability p plays a key role in the performance of the proposed protocol. Having acquired information of the local traffic load, an active node chooses a value of p that optimizes the overall performance of the network. In general, the transmission probability p can be chosen to optimize throughput, power efficiency, or a combination of performance metrics of interest.

For a specific K and known M, the highest overall throughput can be achieved by choosing the value of $p \in [0,1]$ satisfying

$$\frac{\partial R(p;K,M)}{\partial p} = 0,$$

which, with (2), gives

$$\sum_{n=1}^{K} \frac{n\binom{M}{n}p^{n}(1-p)^{(M-n)}(n-pM)}{p\,(1-p)} = 0. \tag{7}$$

However, solving (7) for p analytically is not trivial and, thus, we solve it numerically. This is equivalent to, but potentially less complex than, optimizing (2) over p and leads to smaller values of p than (1) when the traffic is high.

In the following, network performance metrics formulated in Section IV are numerically evaluated for intended traffic λ as

$$\mathrm{E}_{M}\left(Y\big(p_{\mathrm{opt}}(K,M);K,M\big)\right)$$

where Y takes any of the metrics in (2), (3), and (5), $p_{\mathrm{opt}}(K,M)$ satisfies (7), and M follows a Poisson distribution with mean λ. Simulation results are also provided as a complement to the results of numerical evaluations.

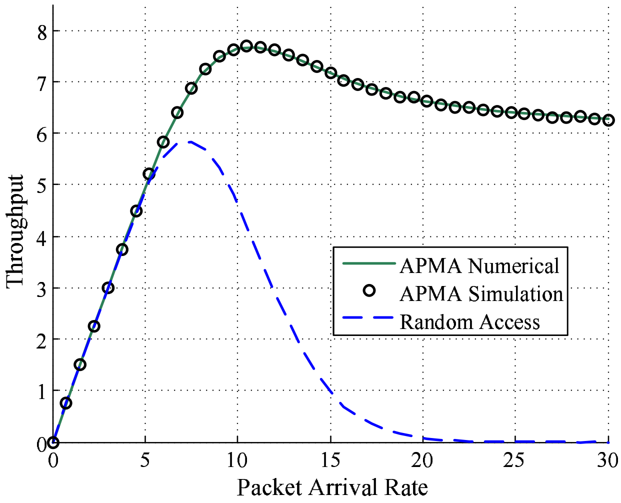

Fig. 1. Adaptive probabilistic medium access vs. random access.

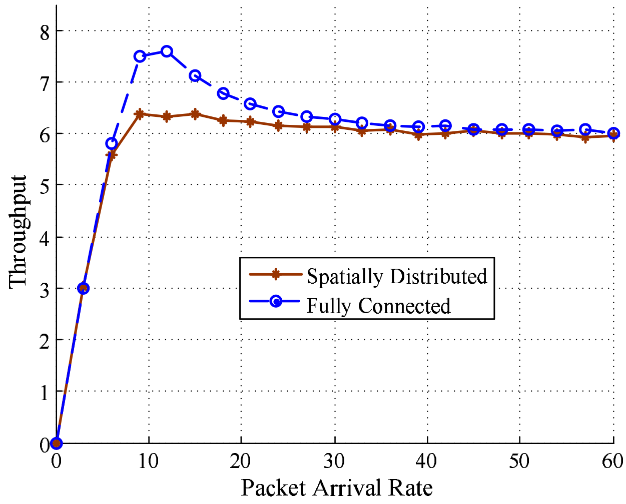

Fig. 2. The effect of node spatial distribution.

A. Adaptive Probabilistic Access vs. Random Access

We compare the average throughput of the proposed adaptive probabilistic access protocol versus Aloha-type random access. Authors in [8] showed that, in terms of average throughput, Aloha-type medium access is asymptotically optimal for the offered traffic $\lambda < K$ as $K \to \infty$. That is, for large K, the average throughput approximately follows the offered traffic as long as it does not exceed K. However, as the offered traffic grows larger than K, the average throughput drops abruptly and packets experience unbounded delay.

Fig. 1 illustrates the performance of the adaptive probabilistic access versus random access for $K = 10$. As can be seen, the advantage of the proposed protocol is two-fold. First, when K is small, adaptive probabilistic access can achieve a higher throughput compared to random access, even when the offered traffic load is optimally adjusted. Second, in the high-traffic regime, where the performance of random access degrades to zero, adaptive probabilistic access keeps the network throughput high.

B. The Effect of Node Spatial Distribution

As discussed, the acquired information about the local offered traffic may not be precise in the transmitter when nodes are spatially distributed. Fig. 2 shows the results of simulating a spatially-distributed network versus a fully-interconnected network such as a typical personal- or body-area network. It can be observed that, with a spatial distribution, performance of the network is almost as good. Nonetheless, the information impreciseness imposes limitations on the network to reach the maximum expected performance.

C. Power Efficiency and Average Packet Delay

The power efficiency and the average packet delay of the proposed protocol were formulated in Section IV. As shown in Fig. 3, the protocol shows a good performance in the high-traffic regime where the power efficiency of random access is practically zero. In wireless sensor networks where efficient battery usage is of essential importance, the network has also the option of power optimization in power-critical areas.

The average packet delay under a Poisson traffic is also shown Fig. 3. As expected, the average delay grows unbounded when the offered traffic becomes large. This effect is unavoidable when the transmission traffic exceeds the capacity of receivers.

VI. Conclusion

Adaptive probabilistic medium access for MPR-capable adhoc wireless networks was proposed. In this protocol, nodes adapt their transmission probabilities according to the local traffic conditions. Analysis of the network performance and simulations showed that the protocol is able to exploit the MPR capability efficiently to achieve high throughput. Although Aloha-type random access is asymptotically optimal for large MPR capability K with the offered traffic not exceeding K, the proposed protocol shows better performance for small K and/or when the traffic is larger than K. The performance of the network in terms of throughput, power efficiency, and delay was analyzed and evaluated via numerical evaluations and simulations.

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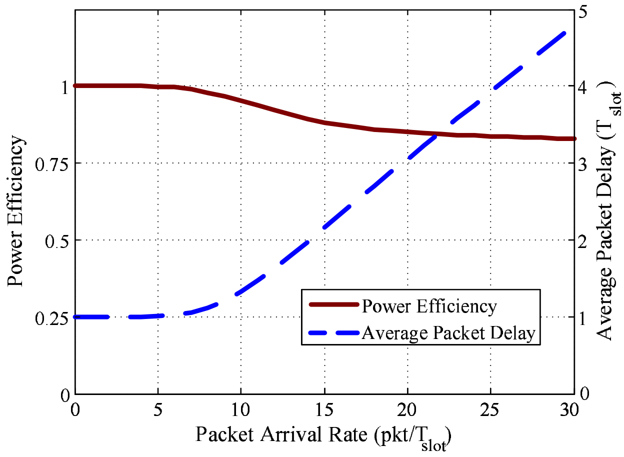

Fig. 3. Power Efficiency and Average Packet Delay.

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APPENDIX

Proof of Lemma 1 Let $A_1 = A_2$ be areas of two disjoint zones of a Poisson point process with density ρ, and let N_1 and N_2 be the numbers of points in the two zones respectively. According to the definition of the Poisson point process,

$$\Pr(N_1 = n) = \Pr(N_2 = n) = \frac{(\rho A)^n \exp(-\rho A)}{n!}, \qquad (8)$$

where $A \doteq A_1 = A_2$. The goal is to calculate the probability distribution function (pdf) of the difference $\Delta N \doteq N_1 - N_2$,

$$P_{\Delta N}(n) \doteq \Pr(\Delta N = n), \quad n = 0, \pm 1, \pm 2, \pm 3, \ldots$$

Due to symmetry between N_1 and N_2 in (8), we have $P_{\Delta N}(n) = P_{\Delta N}(-n)$. Due to disjointness of A_1 and A_2 and the *independence* property of the Poisson point process, we have

$$\begin{aligned} P_{\Delta N}(n) &= \sum_{k=0}^{\infty} \left(\frac{(\rho A)^k \mathrm{e}^{-\rho A}}{k!} \right) \left(\frac{(\rho A)^{(k+|n|)} \mathrm{e}^{-\rho A}}{(k+|n|)!} \right) \\ &= \mathrm{e}^{-2\rho A} \sum_{k=0}^{\infty} \left(\frac{(\rho A)^{2k+|n|}}{k!(k+|n|)!} \right), \end{aligned}$$

and, thus [12],

$$P_{\Delta N}(n) = \mathrm{e}^{-2\rho A} I_{|n|}(2\rho A), \quad n = 0, \pm 1, \pm 2, \ldots \qquad (9)$$

where $I_\alpha(.)$ is the modified Bessel function of the first kind of order α.

Proof of Theorem 2 Let $\rho_{\rm net}$ be the density of the Poisson point distribution of network nodes. As given, each node, independent of other nodes, is active with probability α. Hence, according to the *stochastic multiplexing* property of the Poisson point process, the spatial distribution of the active nodes is also a Poisson point process with density $\rho \doteq \alpha \rho_{\rm net}$.

Let $A_{\rm nbr}$ be the neighborhood area of a node. Consider a transmitter-receiver pair with neighborhood area intersection of A_c. Then, the area hidden from the transmitter A_h and the area exposed to the transmitter A_x are identical. Define $A \doteq A_h = A_x = A_{\rm nbr} - A_c$. Also let N_h and N_x be, respectively, the numbers of active nodes in A_h and A_x. Then, according to lemma 1, the pdf of the difference $\Delta N = N_h - N_x$ follows (9).

Now, we study the asymptotic case as the MPR capability $K \to \infty$. We consider the nontrivial case where the traffic load does not remain constant, but increases proportionally, i.e. $O(\rho A) = O(K)$. Numerical evaluations show that the following expression

$$\sum_{n=-\sqrt{2\rho A}}^{\sqrt{2\rho A}} \exp(-2\rho A) I_{|n|}(2\rho A),$$

is bounded away from 0 and 1 as $\rho A \to \infty$. Therefore, the probability of having an estimate of $O(\rho A) = O(K)$ goes to zero with $O(\frac{1}{\sqrt{\rho A}})$. That is, the probability of a *significant* miscount by the transmitter goes to zero as $K \to \infty$.

Corollary 3: For the case of circular neighborhood areas, the pdf of the miscount follows (9) where

$$\begin{aligned} A &= \pi r^2 - A_c \\ &= r^2 \left(\pi - 2\cos^{-1}\left(\frac{d}{2r}\right) \right) - \frac{1}{2} d \sqrt{4r^2 - d^2}. \end{aligned} \qquad (10)$$

In the above equation, A_c is the intersection of areas of two circles with radii $r_1 = r_2 = r$ whose centers are d distant.

Proof: See [13] for (10). ■